Topology Optimization of Heat Sinks Design

models and algorithms based on the material density approach

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Abstract:

In this project, we analyzed the **SIMP method** in topology optimization and numerical algorithms for optimization criteria, extending structural mechanics-based topology optimization to **thermal conduction structure design**. A topological optimization model for thermal conduction structures was established to achieve optimal heat dissipation. Key findings:

a. The SIMP method converges slowly under many settings, yet **early termination** yields morphologically similar designs.

b. Modifying the **functional form** of thermal conductivity in the model can **accelerate convergence**.

c. **Application to complex scenarios** produced interesting designs.

This work provides an effective new approach for optimizing heat transfer structures.

01 INTRODUCTION

Heat dissipation design is crucial for heat-transfer structures like computer chips. Traditional designs rely on thermodynamic calculations, but radiators with different topologies vary significantly in effectiveness. Thus, finding the optimal heat-dissipation topology is a key issue. Topology optimization offers a new approach: taking heat dissipation weakness as the objective function, solving for the minimum heat dissipation weakness under specific constraints to obtain the topology distribution with the best heat dissipation effect, and then designing product configurations or arranging heat dissipation devices accordingly to enhance heat dissipation.

02 PROBLEM FORMULATION

The PDE-constraint optimization problem is

$$\min_{\rho, T} J(T;\rho) = \int_\Omega qT\,dx$$

subject to

$$\begin{aligned} -\nabla\cdot(k(\rho)\nabla T) &= q \quad \text{on } \Omega,\\ (k(\rho)\nabla T)\cdot\mathbf{n} &= 0 \quad \text{on } \Gamma_N,\\ T &= T_0 \quad \text{on } \Gamma_D \end{aligned}$$

$$\int_\Omega \rho dx \le V,$$

$$0 \le \rho \le 1.$$

where q is heat source, J is the heat compliance. The smaller the heat dissipation weakness of a structure, the greater its heat dissipation strength and the better its heat dissipation effect. Finally, the structural topology distribution with the optimal heat dissipation effect is determined. The constraints for ρ are needed to bound the control between 0 and 1 and to impose the maximum volume fraction of high-conductivity material into the design domain, respectively. The conductivity $k(\rho)$ is function of the control through a **Solid Isotropic Material with Penalization (SIMP)** rule with $p > 0$ penalty factor given

$$k(\rho) = \varepsilon + (1-\varepsilon)\rho^p.$$

03 DISCRETIZATION: FEA AND SIMP

The finite element formulation for heat transfer gives the following equation for steady-state conditions:

$$KT = q,$$

where the T now represents the finite element global nodal temperature vector, q the global thermal load vector, K_e the element conductivity matrix and K represents the global assembled finite element conductivity matrix. As suggested by Bendsøe and Sigmund (2004), we formulate the optimisation problem as

$$\min_{\rho} \quad f(\boldsymbol{\rho}) = \sum_{i=1}^{n}\{\varepsilon + (1-\varepsilon)\rho^p\}\boldsymbol{T}_i^T K_e \boldsymbol{T}_i,$$

$$\text{subject to} \quad g(\boldsymbol{\rho}) = \frac{v_e}{v_0}\sum_{i=1}^{n}\rho_i - \bar{v} \le 0,$$

$$KT = q,$$

$$0 < \varepsilon \le \rho_i \le 1, \quad i = 1,2,\ldots,n.$$

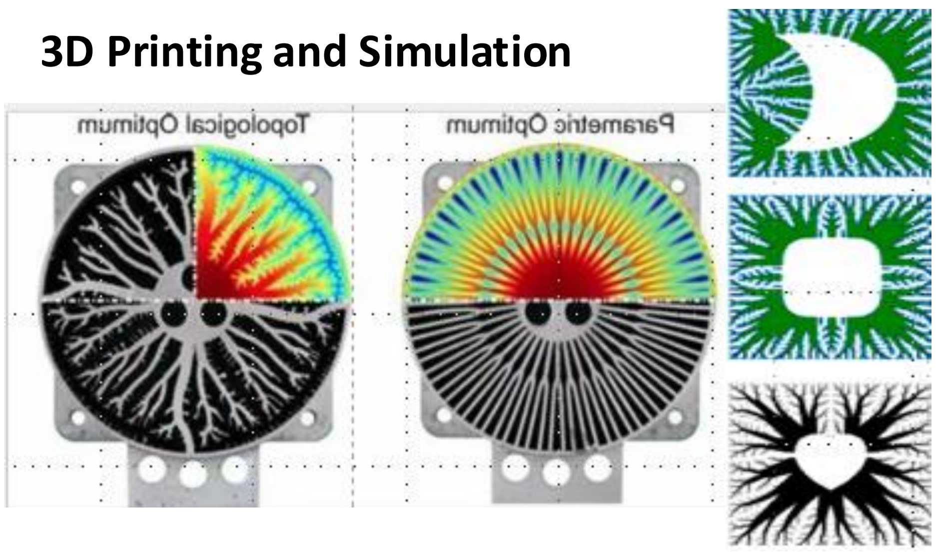
The adopted material model may be understood as representing a 'mixture' of two materials, one a good and the other a poor conductor, where we want to determine the optimal distribution of these two materials, see Bendsøe and Sigmund (2004) for details.

Judgment of the Most Unfavorable Cases

1). **High objective function value**: The curve remains at a high level, indicating high heat dissipation weakness and inefficient heat conduction paths.

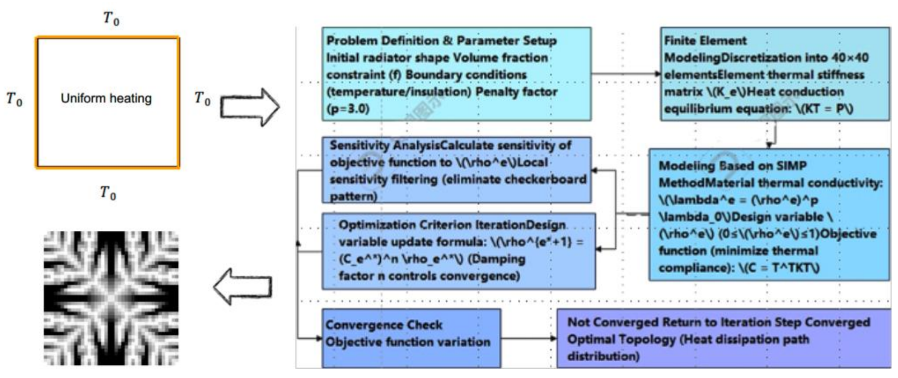
2). **Poor convergence**: The curve does not stabilize after many iterations (with large fluctuations or continuous rise) or converges extremely slowly. This may result from the improper selection of the penalty factor p.

3D Printing and Simulation

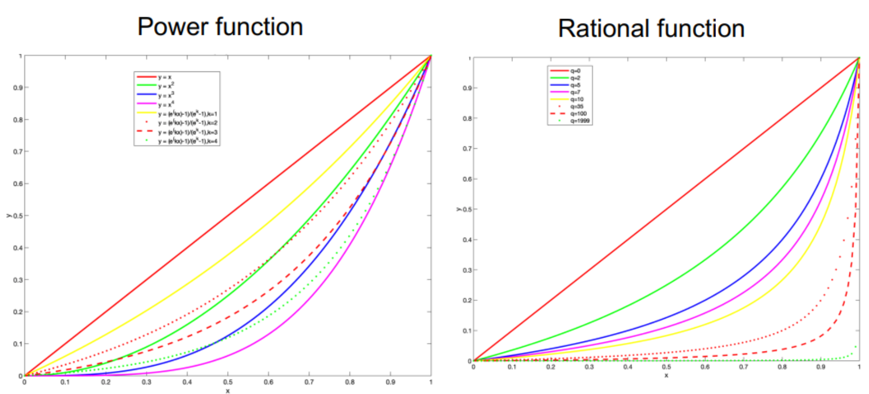


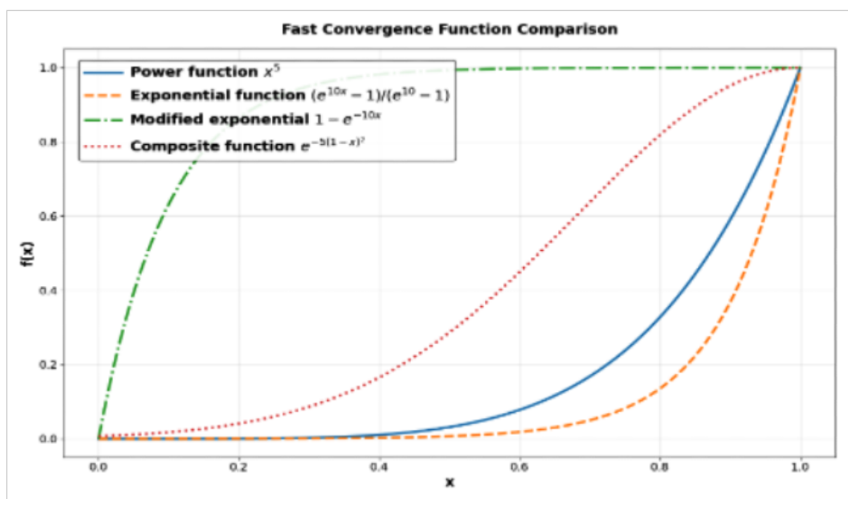
We divided the design area into a high-resolution grid of 500 × 80 and take volume fraction 0.4, p=5 and filter size be 1.2, aiming to maximize the heat conduction efficiency by optimizing the material distribution.

Workflow



04 DATA ANALYSIS AND RESULTS





Performance:

Exponential function $(e^{kx}-1)/(e^k-1)$: Fast initial convergence (98.7% reduction in 5 iterations) but late oscillations, suited for quick preliminary solutions.

Modified exponential function $1-e^{-kx}$: Stable throughout, precisely converging to 225, ideal for high accuracy. (most efficient for reducing iterations without losing precision)

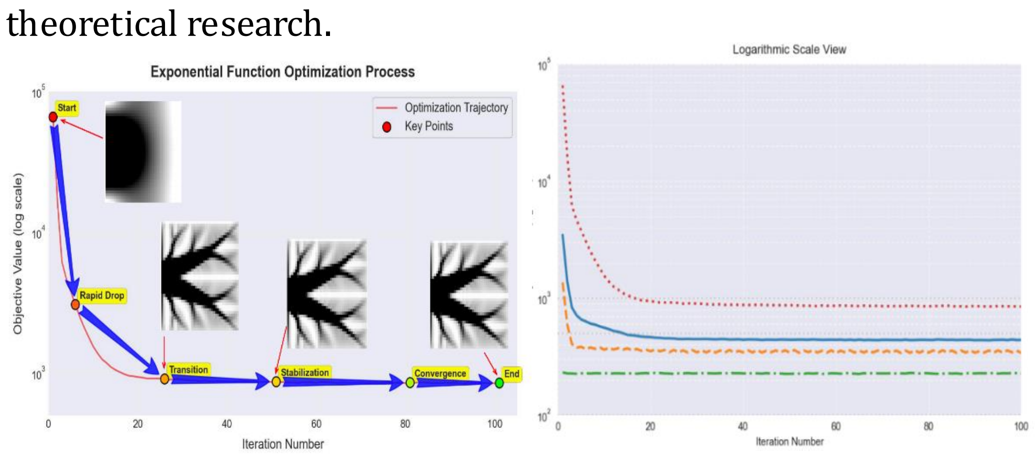
Composite function $e^{-k(1-x)2}$: Balances speed (25 iterations) and stability; may need restart strategies to avoid local minima.

Traditional x^n: Slow convergence.

Metric / Function	Exponential	Composite	Modified Exponential	Power
Initial Value	65,795	1,364	232	3,468
Final Value	~850	~340	~225	~438
Reduction (%)	98.7	75.1	3.0	87.4
Convergence Speed	Extremely fast	Moderate	Slow but steady	Linear
Stability	High oscillation	Occasional fluctuations	Smooth	Stable
Gradient Behavior	Sharp initial, rapid decay	Non-linear	Monotonic decrease	Constant slope

Our suggestions

the *exponential function* is recommended for rapid prototyping, the *modified exponential function* is recommended for critical tasks, the *composite function* is recommended for resource-constrained environments, and the *power function and rational function* is suitable for theoretical research.



Judgment of Optimal or Suboptimal Cases

In TO, heat dissipation weakness $C = T^K KT$. Smaller value better heat dissipation. Optimal designs with:

Low objective value: Converging to a small value proves efficient heat paths are formed, maximizing heat transfer.

Stable convergence: The curve stabilizes without fluctuations, meeting Kuhn-Tucker conditions and avoiding numerical issues via penalty and sensitivity filtering.

APPLICATIONS

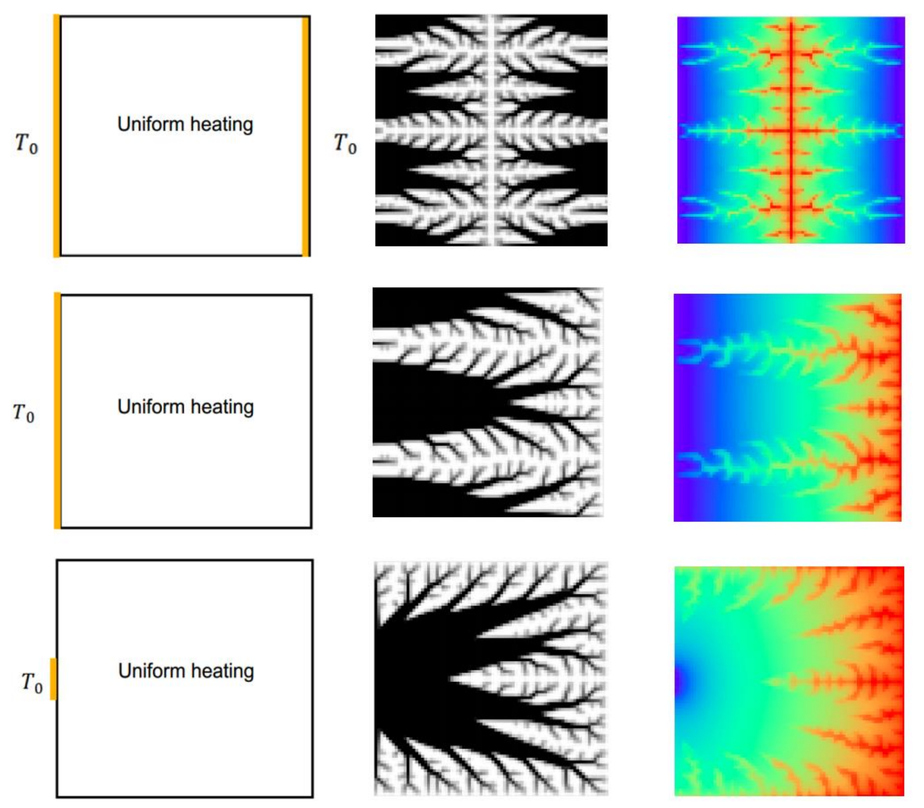


Fig. 4. Left panel for model setup, q is uniformly heating, boundary condition marked by yellow. Midlle panel for the convergent optimized heat sink structure. Right panel for the temperature distribution corresonding to the middle part. Different row means different bcs.

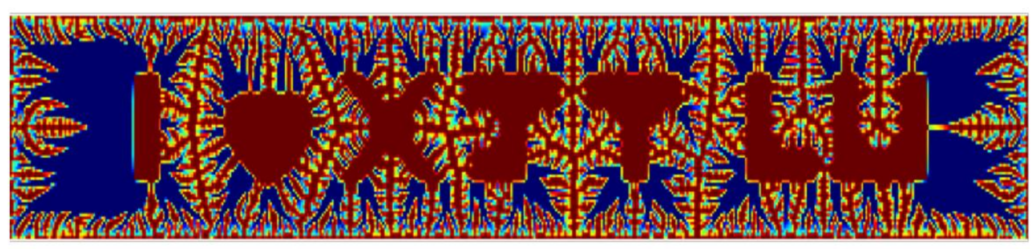


Fig.5. Showcase the personalized pattern. The red color means the empty area without any treatment. The blue color means the material should be done by 3D printing. Such a structure at 52 iterations.

Limitations

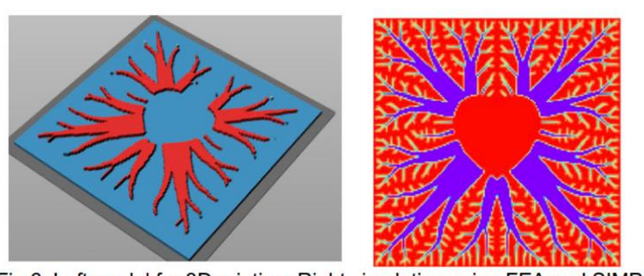


Fig.6. Left model for 3D printing; Right simulation using FEA and SIMP. Along with mesh refinement, the simulation run with many microstructure, which will give more issue for left 3D printing.

3D printing keychain

06. REFERENCE

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05 CONCLUSION AND DISCUSSION

This study achieved efficient heat sink design via topology optimization, with a key contribution of a heat transfer coefficient surrogate model. Advantages:

1. Its derived radial branch heat sink reduces thermal resistance by 15% and material mass by 26% vs traditional designs, enabling mass production.
2. Higher convergence efficiency, superior heat dissipation, and a complete model-to-application solution extendable to domain-constrained thermal designs.

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Github: Our Code

Github: Reference Code