Topic: Physics of Open Systems

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Lecture Notes

The recorder: Changjian Xie 2017/8/10

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Abstract: In these series of lectures, Prof. Zhang introduced a general theory of open systems that they developed in the last 10 years. The fundamental physics of open quantum systems, including topics of nonequilibrium dynamics, non-Markovianity memory phenomena, entanglement decoherence and decoherence of topological states, quantum thermalization, quantum phase transition, and quantum transport in mesoscopic physics are explored. Applications to various open systems are also presented.

At the start of the lecture, He spent one hour to introduce the big picture of the Open Systems, and in the afternoon, he also spent three hours to talk about the work that his group had done.

The outline of his lecture is as follows,

1. General open quantum systems (OQS)
2. A general theory of OQS
3. General analytical solution of non-Markovianity
4. Measurement of non-Markovianity (a new point of view)
5. Non-Markovianity nonequilibrium physics
6. Non-Markovianity entanglement decoherence
7. Non-Markovianity decoherence of Majorana qubits
8. Transient quantum transport and quantum control
9. Summary

Topic 1 - General OQS

Prof. Zhang introduced the background of the OQS and gived the definition. The big picture which be similar as the one that wikipedia does is as follows, in physics, an open quantum system is a quantum-mechanical system which interacts with an external quantum system, the environment. In reality, no quantum system is completely isolated from its surroundings, thus quantum system is open to some extent, causing dissipation in the quantum system. Techniques developed in the context of open quantum systems have proven powerful in fields such as quantum optics, quantum measurement theory, quantum statistical mechanics, quantum information science, quantum thermodynamics, quantum cosmology and semi-classical approximations.

Quantum system and environment means as follows. No quantum system can be completely isolated from its environment. As a direct result a quantum system is never be a pure state. A pure state is unitary equivalent to a zero temperature ground state forbidden by the third law of thermodynamics. A complete description of a quantum system requires to include the environment. The outcome of this process of embedding is that ultimately we end with the state of whole universe described by a wavefunction Ψ.

Even if the combined system is pure state and can be described by a wavefunction Ψ, a subsystem in general cannot be described by a wavefunction. This observation motivated the formalism of density matrices or density operators introduced by John von Neumann and independently, but less systematically by Lev Landau in and Felix Bloch. In general, the state of a subsystem is described by the density operator ρ and an observable by the scalar product $(\rho \cdot \mathbf{A}) = Tr\{\rho\mathbf{A}\}$. There is no way to know if the combined system is pure from the knowledge of the observables of the subsystem. In particular if the combined system has quantum entanglement, the system state is not a pure state.

Open quantum system dynamics means as follows. The theory of open quantum systems seeks an economical treatment of the dynamics of observables that can be associated with the system. Typical observables are energy and quantum coherence. Loss of energy to the environment is termed quantum dissipation. Loss of coherence is termed quantum decoherence. The environment we wish to model as part of our open quantum system is typically very large, making exact solutions impossible to calculate. The reduction problem is difficult therefore a diversity of approaches have been pursued. A common objective is to derive a reduced description where the system's dynamics is considered explicitly and the bath is described implicitly.

The main assumption is that the entire world is a large closed system, and therefore, time evolution is governed by a unitary transformation generated by a global Hamiltonian. For the combined system bath scenario the global

Hamiltonian can be decomposed into

$$H = H_S + H_B + H_{SB}$$

where H_S is the system's Hamiltonian, H_B is the bath Hamiltonian and H_{SB} is the system-bath interaction. The state of the system is obtained from a partial trace over the combined system and bath $\rho_S(t) = Tr_B(\rho_{SB}(t))$. When the interaction between the system and the environment is weak a time dependent perturbation theory seems appropriate. The typical assumption is that the system and bath are initially uncorrelated $\rho(0) = \rho_S \otimes \rho_B$. The idea has been originated by Felix Bloch and followed by Redfield known as the Redfield equation. Redfield equation is a Markovian master equation that describes the time evolution of the density matrix ρ . The drawback of the Redfield equation is that it does not conserve the positivity of the density operator.

An alternative derivation employing projection operator techniques is known as the Nakajima-Zwanzig equation. The derivation highlight the problem that the reduced dynamics is non-local in time

$$\partial_t \rho_{\mathrm{S}} = \mathcal{P}\mathcal{L}\rho_{\mathrm{S}} + \int_0^t dt' \mathcal{K}(t')\rho_{\mathrm{S}}(t-t').$$

The effect of the bath is hidden in the memory kernel $\kappa(\tau)$. Additional assumptions of a fast bath are required to lead to a time local equation $\partial_t \rho_S = \mathcal{L}\rho_S$.

Another approach emerges as an analogue of classical dissipation theory developed by Ryogo Kubo and Y. Tanimura. This approach is connected to Hierarchical equations of motion which embed the density operator in a larger space of auxiliary operators such that a time local equation is obtained for the whole set and their memory is contained in the auxiliary operators.

A formal construction of a local equation of motion with a Markovian property is an alternative to a reduced derivation. The theory is based on an axiomatic approach. The basic starting point is a completely positive map. The assumption is that the initial system-environment state is uncorrelated $\rho(0) = \rho_S \otimes \rho_B$ and the combined dynamics is generated by a unitary operator. Such a map falls under the category of Kraus operator. The most general type of and time-homogeneous master equation with Markovian property describing non-unitary evolution of the density matrix that is trace-preserving and completely positive for any initial condition is the Gorini-Kossakowski-Sudarshan-Lindblad equation or GKSL equation

$$\dot{\rho}_S = -\frac{i}{\hbar}[H_S, \rho_S] + \mathcal{L}_D(\rho_S)$$

H_S is a Hamiltonian part and $\mathcal{L}_D$,

$$\mathcal{L}_D(\rho_S) = \sum_n \left(V_n \rho_S V_n^\dagger - \frac{1}{2}\left(\rho_S V_n^\dagger V_n + V_n^\dagger V_n \rho_S\right)\right)$$

is the dissipative part describing implicitly through system operators V_n the influence of the bath on the system. The Markov property imposes that the system and bath are uncorrelated at all times $\rho_{SB} = \rho_S \otimes \rho_B$. The GKSL equation is unidirectional and leads any initial state ρ_S to a steady state solution which is an invariant of the equation of motion $\dot{\rho}_S(t \to \infty) = 0$. The family of maps generated by the GKSL equation forms a Quantum dynamical semigroup. In some fields such as quantum optics the term Lindblad superoperator is often used to express the quantum master equation for a dissipative system. E.B. Davis, derived the GKSL with Markovian property master equations using perturbation theory thus fixing the flaws of the Redfield equation.Davis construction is consistent with the Kubo-Martin-Schwinger stability criterion for thermal equilibrium, i.e., the KMS state.

Prof. zhang talked about Dirac equation $(i\gamma\cdot\partial - m)\psi = 0$. This is content of the theory of field. We introduce a new field and note that

$$\begin{aligned}\int \mathcal{L} = \int \{&\bar{\psi}(i\gamma\cdot\partial - m)\psi - g_\alpha \bar{\psi}\gamma\cdot A^\alpha \psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &+ |(\partial_\mu + ig' A_\mu)\phi|^2 - \frac{\lambda^2}{4}(|\phi|^2 - \nu^2)^2 + \frac{1}{16\pi}(R - 2\Lambda)\}\sqrt{-g}\, d^4x\end{aligned}$$

where α is the parameter which describes the interaction of different substances, such as strong interaction, weak interaction, etc. As we all know, electron interaction is easier than strong interaction. He looked forward to the new physics. In quantum mechanics, we turn attention to energy instead of the force, we research force in the past, such as $F = ma$, and now, we pay attention to energy, such as $E = \hbar\omega$. We will spend our time researching the information in the future, such as $S = -tr(\rho \ln \rho)$. The information means entropy, i.e., the extent of chaos, from the variation, it follows that

$$\rho = \frac{1}{Z}\exp\{-\beta(H - \mu N)\}$$

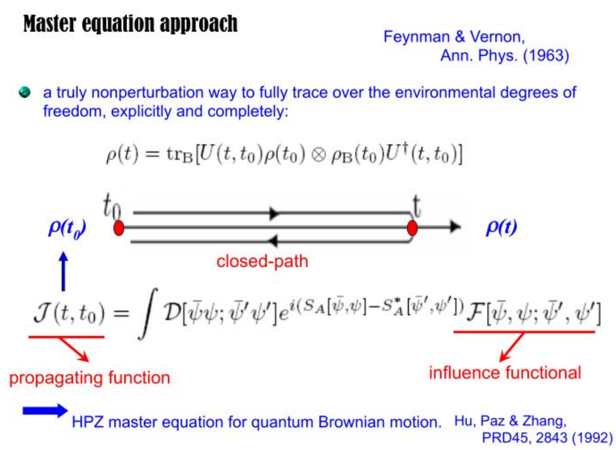

Figure 1: Master equation approach

We also note the balance state $tr(\rho) = 1$, $< H >=$ const, $< N >=$ const. He also talk about nonequilibrium statistics of open systems. All physics which come from the nonequilibrium state are open. As for $S = -tr[\rho \ln \rho]$, and we should understand the $\rho(t)$ from the fundamental view of point. the information is unknown. It happens information exchange, matter exchange and energy exchange between the system and environment.

The essence of closed and open systems

① For closed systems quantum states, i.e., $S = 0$, there's no information.

$$i\hbar \frac{d}{dt}|\psi(t) >= H|\psi(t) >,$$

this is from Schwinger equation.

In terms of density matrix $\rho(t) = |\psi(t) >< \psi(t)|$, we note that

$$\frac{d}{dt}\rho(t) = \frac{1}{i\hbar}[H, \rho(t)],$$

this is from the Von-Neumann equation and Landau.

② For open systems quantum states in general, i.e., $S \neq 0$, there's rich information. Note that $\rho(t) \neq |\psi(t) >< \psi(t)|$, this means mixed state, i.e., the loss of coherence. Meanwhile, we don't know the equation $i\hbar\frac{d}{dt}\rho(t)$ equals what. He gives the physics significance of open systems as follows,

(a) quantum foundation, e.g., origin of probability.

(b) quantum information and computation, i.e., decoherence.

(c) quantum thermodynamics mesoscopic scale.

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He also gived us some examples of typical open systems, and showed the modeling OQS. The most attention should be focused on the research of dynamics evolution. As for the OQS, the control, environment and measurement play an important role in this research region. For example, we can also find the open system by the renormalized group in the H-space.

Application to quantum transport in mesoscopic physics

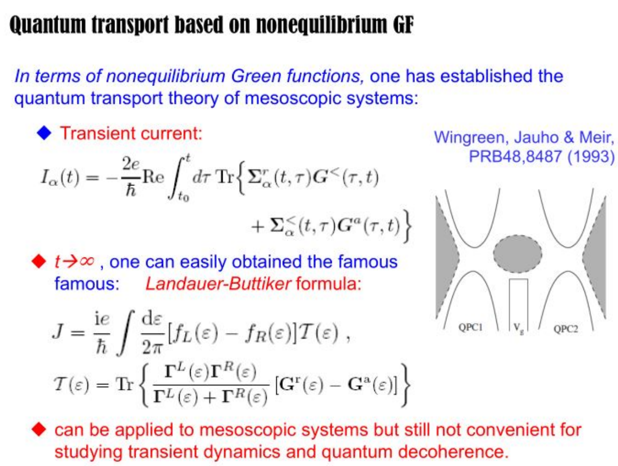

Figure 2: Quantum transport based on nonequilibrium GF

In terms of the Schwinger-Keldysh Green functions, the transport current in mesoscopic physics is given by

$$I_\alpha(t) = -\frac{2e}{\hbar}Re\int_{t_0}^{t} d\tau \mathrm{Tr}\{\sum\nolimits^r_\alpha(t,\tau)G^<(\tau,t) + \sum\nolimits^<_\alpha(t,\tau)G^a(\tau,t)\},$$

we take $t\to\infty$, it gives the Meir-Wingreen formula

$$J = \frac{e}{\hbar}\int\frac{d\varepsilon}{2\pi}[f_L(\varepsilon) - f_R(\varepsilon)]T(\varepsilon),$$

where

$$T(\varepsilon) = tr[G^a\Gamma^R G^r\Gamma^L] \rightarrow tr[tt^{\uparrow}(\varepsilon)]$$

A microscopic expression of Landauer formula is also presented on the screen.

Building the theory of Open System

Theory of OQS is a long-standing problem in general. We note firstly that the master equation

$$\frac{d}{dt}\rho(t) = -\frac{i}{\hbar}[H_s, \rho(s)] + \int_{t_0}^{t} \mathrm{d}_\tau \mathscr{L}(t-\tau)\rho(\tau),$$

the left hand term and the first term of right hand merge to the quantum mechanics from the Von-Neumann and Landau, the second term of the right hand is from Non-Markovian memory. This is non-equilibrium physics. If a system has memory, it should be the open system.

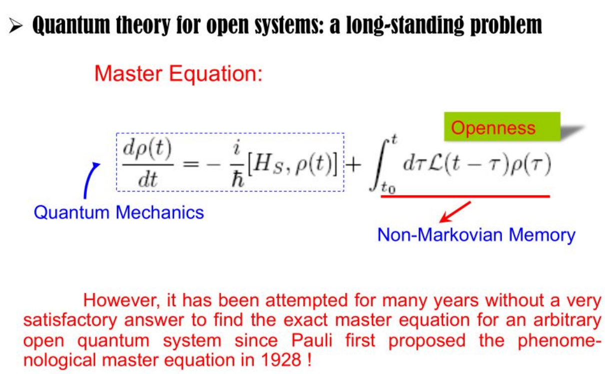

Figure 3: Master Equation

A master equation is a phenomenological set of first-order differential equations describing the time evolution of the probability of a system to occupy each one of a discrete set of states with regard to a continuous time variable t. The most familiar form of a master equation is a matrix form

$$\frac{d\vec{P}}{dt} = \mathbf{A}\vec{P},$$

where $\vec{P}$ is a column vector (where element i represents state i), and $\mathbf{A}$ is the matrix of connections. When the connections depend on the actual time

$$\frac{d\vec{P}}{dt} = \mathbf{A}(t)\vec{P}.$$

When the connections represent multi exponential jumping time probability density functions, the process is semi-Markovian, and the equation of motion is an integro-differential equation termed the generalized master equation

$$\frac{d\vec{P}}{dt} = \int_0^t \mathbf{A}(t-\tau)\vec{P}(\tau)d\tau.$$

The Lindblad equation in quantum mechanics is a generalization of the master equation describing the time evolution of a density matrix. Another special case of the master equation is the Fokker-Planck equation which describes the time evolution of a continuous probability distribution.

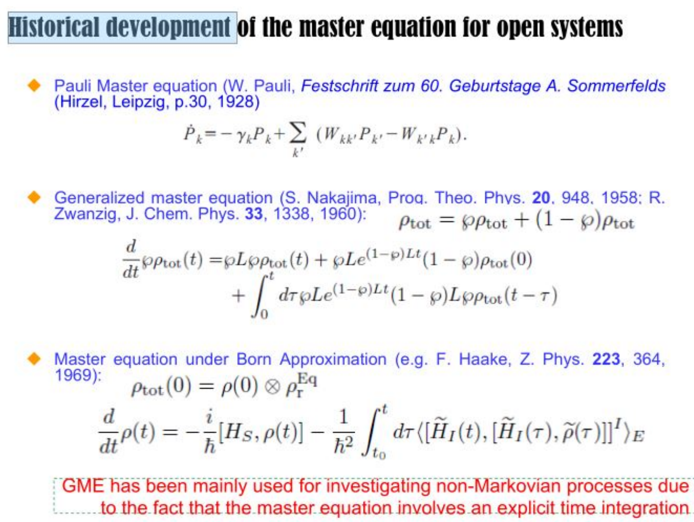

Figure 4: Historical development of the master equation for open systems

Finally, it develop into Born-Markov equation. As a matter of fact, the application of the Lindblad-GKS form of an approximate master equation is quite wide due to the symmetry and simplicity, and it refers to the dissipation. As for

$$\frac{d}{dt}\rho(t) = \frac{1}{i}[H_s(t)\rho(t)],$$

we pay attention to the dissipation, fluctuations and Non-Markovian memories. Then, goes beyond the Markovian dynamics. He focuses on the three timescale dominate the dynamics of Open System.

The motivation is as above. In the sequel, he talked about his work.

Topic 2 - A general theory of OQS

Prof. Zhang introduced two useful theories for nonequilibrium dynamics. One is the Schwinger-Keldysh's Green function technique. Another one is the Feynman-Vernon influence functional approach. This is for the closed-path approach for nonequilibrium dynamics.

Nonequilibrium Green-function approach

The slide present the Kadanoff-Baym equation as follows,

$$
\begin{aligned}
&\{i\frac{\partial}{\partial\tau_1} - [-\frac{1}{2}\nabla^2_{x_1} + U(x_1,\tau_1)]\}G(x_1,\tau_1,x_{1'},\tau_{1'}) \\
&\qquad = \delta(1-1') + \int_C d\sigma \int d^3y \sum(x_1,\tau_1,y,\sigma)G(y,\sigma,x_{1'},\tau_{1'})
\end{aligned}
$$

where L.V.Keldysh is as follows,

$$
\begin{pmatrix} G^T & G^< \\ G^> & G^T \end{pmatrix} \longrightarrow \begin{pmatrix} G^c & G^r \\ G^a & 0 \end{pmatrix}
$$

where

$$
\begin{aligned}
G^r(t,t') &= \theta(t-t')(G^> - G^<), \\
G^a(t,t') &= \theta(t'-t)(G^< - G^>), \\
G^c(t,t') &= G^> + G^<.
\end{aligned}
$$

It follows that

$$
i\frac{d}{d\tau} - \omega G^r(\tau,\tau') = \delta(\tau-\tau') + \int_{\tau'}^{\tau} \sum{}^r(\tau,\tau'')G^r(\tau'',\tau')\,d\tau''
$$

$$
G^< = (1 + G^r\sum{}^r)G_0^<(1 + \sum{}^a G^a) + G^r\sum{}^< G^a
$$

where the $G^<$ is lesser GF and G^r is the retarded GF.

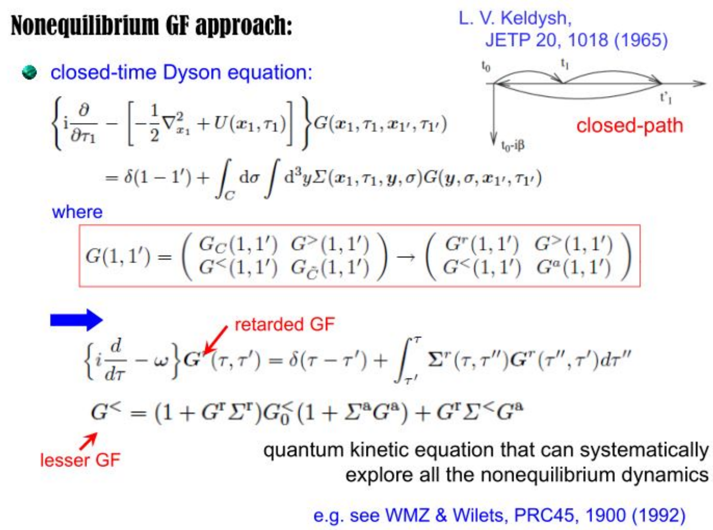

Figure 5: Nonequilibrium GF approach

The correlation function or the evolution of density matrix is as follows,

$$G(x,t;x't') = -i < T_c|\psi(xt)\psi^\dagger(x't')| > \quad \text{if } t > t'$$

$$\rho(t) = U(t,t_0)\rho(t_0)U^\dagger(t,t_0).$$

The Feynman-Vernon's approach is common useful. He also talk about the limitation of quantum Brownian motions. The system-environment coupling for Brownian motions is as follows,

$$H_{\text{int}} = x\sum_k c_k q_k = \sum_k c_k^{'}(a^\dagger b_k + ab_k^\dagger + a^\dagger b_k^\dagger + ab_k),$$

where a and b_k is known. And the last two term of right hand is particle pair production.

Quantization of classical system is not always correct. Our consideration for general system-environment couplings (generalized Fano-Anderson Models) is as follows,

$$H_{\text{SB}} = \sum_{\alpha k i}[V_{\alpha k i}a_i^\dagger + V_{\alpha k i}^* b_{\alpha k}^\dagger a_i]$$

$$\mathcal{J}_{\alpha ij}(\omega) = 2\pi\sum_k V_{\alpha ik}V_{\alpha jk}^*\delta(\omega - \varepsilon_k),$$

the second equation as above is special density.

System-Environment coupling II

If the system and environment couple for the same kind of particle, we obtain the representation of H_{SB} and H_{e-p}. We focus on

$$H_{\mathrm{SB}} = \sum_{\alpha k i}[V_{\alpha k i} a_i^\dagger b_{\alpha k} + V_{\alpha k i}^\dagger b_\alpha^\dagger a_i]$$

Only it be continuous, the system will be open system. Prof. Zhang introduced the integrating over all environment degrees of freedom as well. He gived a few crucial steps toward obtaining the master equation. Moreover, He deduced the exact master equation as follows,

$$\begin{aligned}\frac{d}{dt}\rho(t) = \frac{1}{i}[\tilde{H}_s(t), \rho(t)] + \sum_{ij}\{r_{i,j}(t)[2a_j\rho(t)a_i^\dagger - a_i^\dagger a_j \rho(t) - \rho(t)a_i^\dagger a_j] \\ + \tilde{r}_{i,j}(t)[a_i^\dagger \rho(t) a_j \pm a_j \rho(t) a_i^\dagger \mp a_i^\dagger a_j \rho(t) - \rho(t) a_j a_i^\dagger]\},\end{aligned}$$

where the terms containing the $r_{i,j}(t)$ is dissipation term, and the terms containing $\tilde{r}_{i,j}(t)$ is the fluctuation term.

Dissipation and fluctuation coefficient in the master equation is $\tilde{\varepsilon}(t)$, $\gamma(t)$ and $\tilde{\gamma}(t)$ represent by u and v, where the $\tilde{\varepsilon}(t)$ is called renormalization. He gived the nonequilibrium Green function $u(t, t_0)$ and $v(t, t)$. He introduced the equation of motion for nonequilibrium Green function and find the connection of master equation with operators. Moreover, He rebuilt the NEGF transport theory. He spent plenty of time to illustrate the master equation. This is the main trick of his lecture. The slide display the Lindblad-GKS form of exact master equation. the exact master equation can be rewritten as another special form, and he went beyond the fundamental theories for Open System. He revised the two methods, i.e., the Schwinger-Keldyshs Green function technique and Feynman-Vernon influence functional approach.

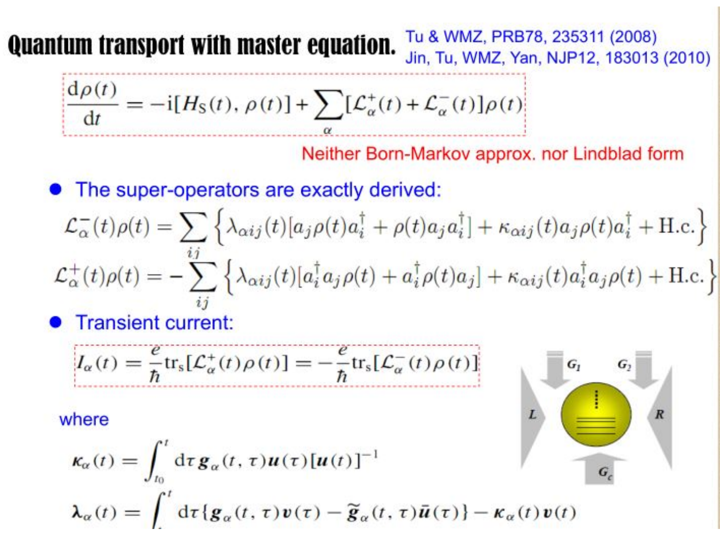

Figure 6: Quantum transport with master equation

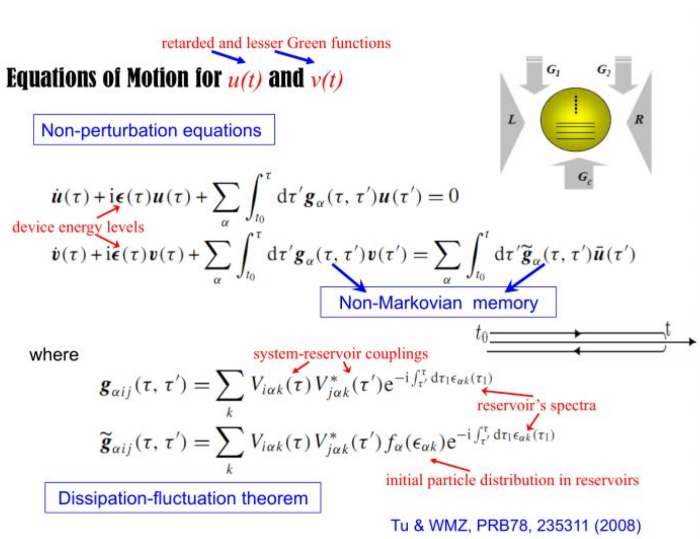

Figure 7: Equations of Motion for $u(t)$ and $v(t)$

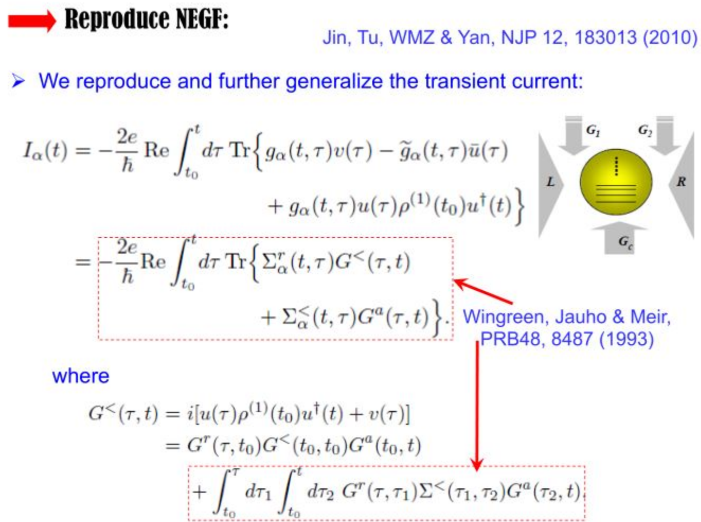

Figure 8: Reproduce NEGF

For partition-free initial states involving initial correlation. Note that

$$\rho_{t_0t}(t_0) = \frac{1}{Z}\exp\{-\beta H_{t_0t}\}.$$

Topic 3 - General analytical solution of non-Markovianity

Prof. Zhang introduced the Non-Markovian dissipation and fluctuations. The slide also presents the universality of Non-Markovian dissipation. He also recommended a textbook that Peskin write called An Introduction to Quantum Field Theory. This is a quite simple for beginner to get started. He also presented the universality of two-point Green function and talked about the universality of non-Markovianity fluctuations. He illustrated the methods to measure the dissipation and fluctuation. There are noise spectrum and spectral density for particle-particle correlation in the slide. The example is bosonic open systems. The general reservoir $\mathcal{J}(\omega)$ and general solution $u(t)$ is presented.

Markovian and Non-Markovian processes

A boson (single-mode photon) system coupled to a thermal bath. He talked about the dissipation to dissipationless transition for Ohmic-type spectral densities.

Some Remarks

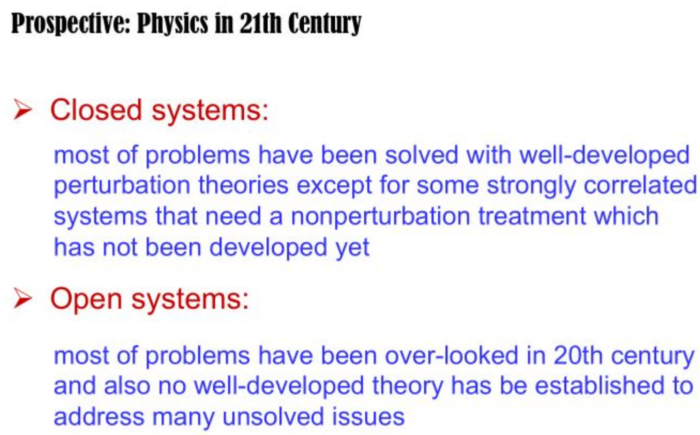

Figure 9: Prospective

The spin system is continuous, thus, it's the OQS. So, many problems in spin system also adopt the series theory of master equation to solve. The trick for this is interesting as well. As we all know, the dissipation and fluctuation are both important for OQS. But the latter is more crucial that the former. I.e., the fluctuation dominates the OQS. The lecture mainly talks some classical theory and methods of OQS, and Prof. Zhang emphasize the importance of entropy in the future. But the least clear concept in entire thermodynamics is entropy as mentioned by Landau. Of course, the entropy is researched quite a lot in the area of biology. The lecture doesn't over, Professor WM.Zhang only talk about the three parts of this topic, the remained six parts isn't presented due to time constraints. The notes only containing the first three parts.

Useful links

The second link is the content of the lecture slide I attended, and I have the audio of the lecture in my personal computer. If you need, please contact me, my email address: 20164207002@stu.suda.edu.cn

1. `http://www.phys.nthu.edu.tw/seminar/AMO/2011F/talk-NTHU(2011-11-17).pdf`

2. `http://web.phys.ntu.edu.tw/colloquium/Joint%20colloquia_files/Slides/20170613.pdf`